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AUTHOR(S):

Ikegami, Hidetsugu; Sano, Mitsuo

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## Effect of Quasi-holes on the Structure of Odd-mass Nuclei

Hidetsugu IKEGAMI\* and Mitsuo SANO\*\*

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It is well known that the Tamm-Dancoff approximation (TDA) is less adequate than the random phase approximation (RPA) for the treatment of the quadrupole and octupole collective vibration in even-even nuclei since the latter includes the ground state correlation.<sup>1)</sup> Although in previous theories RPA has been employed for the description of the even-even core, essentially only the TDA scheme has been used for the odd nucleus since quasi-hole, or the backward scattering effect for an unpaired nucleon, was not taken into account. In the present paper, it is shown that a description including this effect explains some important features of odd nuclei not predicted by previous theories.<sup>2,3)</sup>

We introduce the Hamiltonian and the coupled equations of motion of quasi-particles and quasi-holes. The Hamiltonian is

$$H = H_{\text{BCS}} + (H_{22} + H_{40} + H_{31} + \text{c.c.}) \quad (1)$$

where the operators have been ordered in normal form, i.e., all the creation operators  $\alpha_{jm}^+$  are placed to the left of the destruction operators  $\alpha_{j-m}$ . The indices  $n_1$  and  $n_2$  of  $H_{n_1 n_2}$  refer to the numbers of  $\alpha_{jm}^+$  and  $\alpha_{j-m}$ , respectively. The equation of motion becomes<sup>4)</sup>

$$[H, \alpha_{jm}^+] = E_j \alpha_{jm}^+ + \sum_{j'} K_{jj'} (\lambda \mu j' m' | jm) \alpha_{j'm'}^+ \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ + \sum_{j'} M_{jj'} (\lambda \mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \quad (2a)$$

$$[H, (-)^{j-m} \alpha_{j-m}] = -E_j (-)^{j-m} \alpha_{j-m} - \sum_{j'} K_{jj'} (\lambda \mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ + \sum_{j'} M_{jj'} (\lambda \mu j' m' | jm) \alpha_{j'm'}^+ \{Q_{\lambda\mu}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \quad (2b)$$

where<sup>5)</sup>  $K_{jj'} \equiv -S_{\lambda}^{'-1/2} (2\lambda + 1/2j + 1)^{1/2} \langle j' \| q_{\lambda} \| j \rangle v_{jj'}$

$$M_{jj'} \equiv -S_{\lambda}^{'-1/2} (2\lambda + 1/2j + 1)^{1/2} \langle j' \| q_{\lambda} \| j \rangle u_{jj'}$$

Here  $E_j$  is energy of a quasi-particle with spin  $j$  and  $Q_{\lambda\mu}^+$  ( $Q_{\lambda-\mu}$ ) represents the creation (destruction) operator of a phonon of multipole order  $\lambda$ ,  $\mu$  for the even-even core. It is assumed that  $Q_{\lambda\mu}^+$  and  $Q_{\lambda-\mu}$  commute with  $\alpha_{jm}^+$  and  $\alpha_{j-m}$ , also that  $Q_{\lambda\mu}|0\rangle = 0$ . The physical meaning of the second assumption is: the ground state  $|0\rangle$  of the even-even core in an odd nucleus is the same as the ground state of its even-even neighbor.  $S_{\lambda}^{'-1/2}$ , the strength of the phonon-quasi-particle and phonon-quasi-hole interactions, is calculated in the scheme of RPA (hereafter denoted  $S_{\lambda}^{'-1/2}(\text{e-e})$ ).<sup>2)</sup> In the present cal-

\* 池上栄胤: Research Center for Nuclear Physics, Osaka University, Yamadakami, Suita, Osaka.

\*\* 佐野光男: Department of Physics, Osaka University, Machikaneyamacho, Toyonaka, Osaka.

culution, the BCS equations were solved for two major shells employing level energies which are almost the same as those found from Mottelson, Nilsson, and Prior's orbits ( $\delta=0$ ).<sup>6)</sup> The strengths of the pairing and quadrupole forces were determined to fit the energy gaps obtained from the even-odd mass difference and the energies of the first  $2^+$  states in neighboring even-even nuclei, respectively. For one quasi-particle and hole (in the two major shells) with zero, one, and two phonons, the equations of motion are linearized by employing the following relations<sup>7)</sup>

$$\begin{aligned}
& \langle jm | [H, \alpha_{jm}^+] | N, JM \rangle \simeq (E_j^0 - N\hbar\omega_\lambda) \langle jm | \alpha_{jm}^+ | N, JM \rangle \\
& \langle jm | \{Q_{2\mu}^+ + (-)^{-\mu} Q_{2-\mu}^-\} \alpha_{j'm'}^+ | N, JM \rangle \\
= & \sum_{N', J'M'} \langle jm | \alpha_{j'm'}^+ | N', J'M' \rangle \langle N', J'M' | \{Q_{2\mu}^+ + (-)^{-\mu} Q_{2-\mu}^-\} | N, JM \rangle
\end{aligned}$$

Here  $|jm\rangle$ ,  $E_j^0$ , and  $|N, JM\rangle$  represent a state with spin  $j$  in odd nucleus, its energy eigenvalue, and the  $N$ -phonon state with spin  $J$ , respectively. In the second equation the last term on the right hand side is calculated in the same manner as for an even-even nucleus.

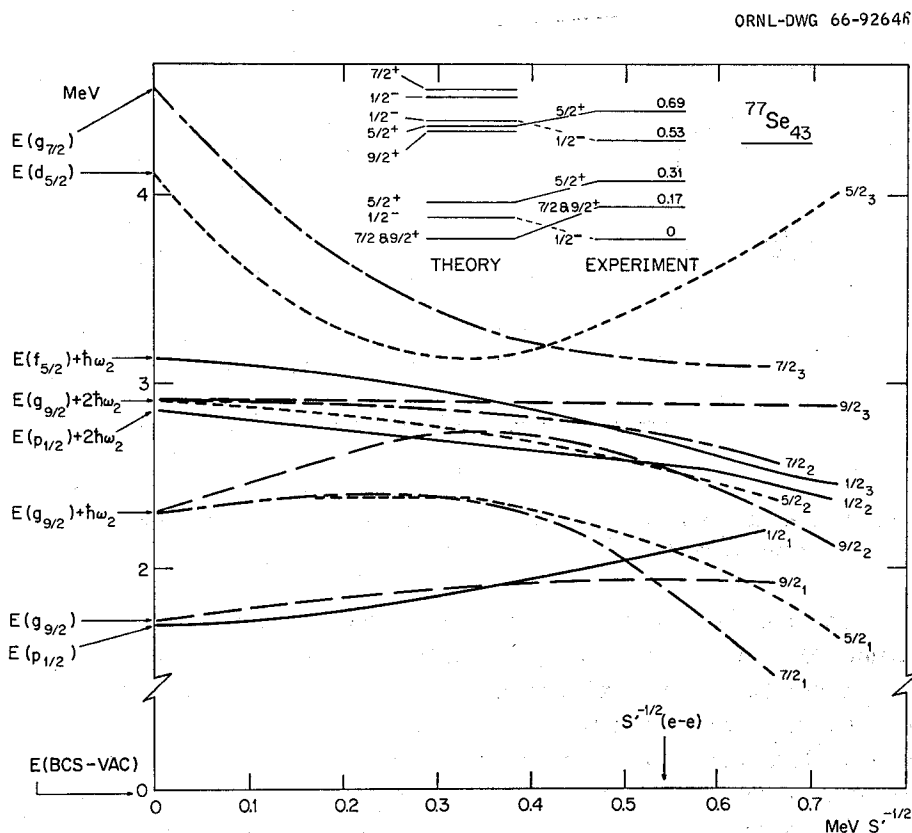


Fig. 1. Dependence on  $S_2^{-1/2}$  of the level ordering for  $\text{Se}^{77}$  calculated taking account of the quasi-hole effect. The value of  $S_2^{-1/2}$  ( $e-e$ ) is estimated from the  $2^+$  level energies of  $\text{Se}^{76}$  and  $\text{Se}^{78}$ .

Figure 1 shows the predicted level ordering in  $^{77}\text{Se}_{43}$  as a function of the strength  $S'^{-1/2}$ . Figure 2 shows the level ordering when the quasi-hole effect is omitted.<sup>8)</sup> The striking features evident from comparison of the two figures are:

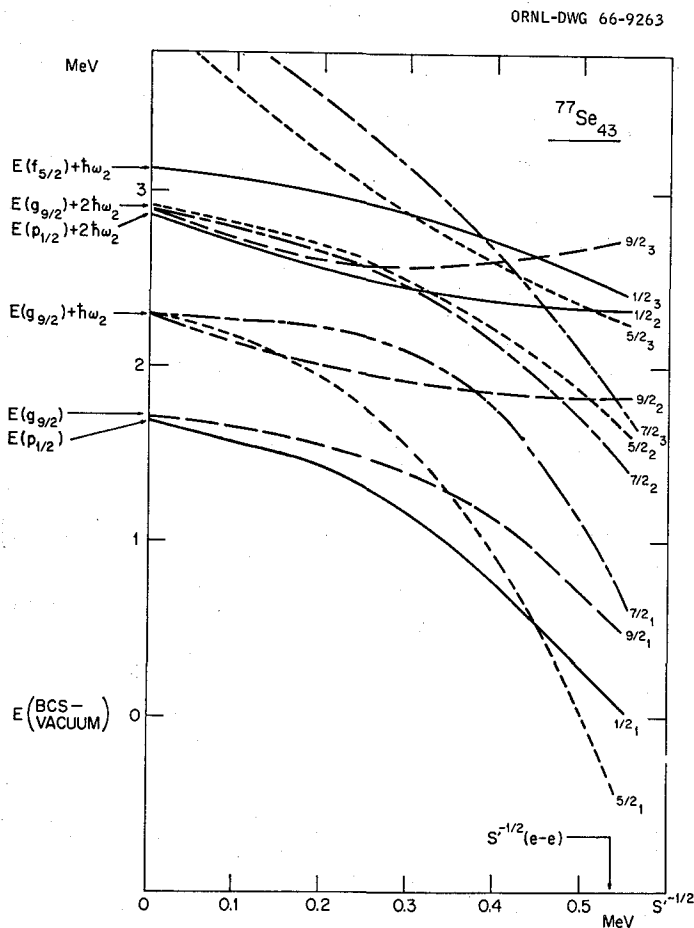


Fig. 2

Fig. 2. Dependence on  $S'^{-1/2}$  of the level ordering for  $^{77}\text{Se}$  calculated neglecting the quasi-hole effect.

i) Levels of quasi-particle near the Fermi level are pushed up because of their interaction ( $M'_{ij}$  term in Eq. 2) with the quasi-holes.

ii) Levels having the same spin and parity lie fairly close together in Fig. 1 in contrast with their separated positions in Fig. 2. The close spacing of Fig. 1 is consistent with observation, close to the ground state of odd nuclei with strong vibrational nature, of a number of doublets having the same spin and parity.<sup>9)</sup>

iii) The pushing-up effect, due to the quasi-hole, on the so called anomalous-coupling  $7/2^+$  and  $5/2^+$  levels is weaker than for normal states. This is another reason for the low-lying position of the anomalous-coupling states in addition to our previous explanation.<sup>6)</sup>

iv) Though the effect of the quasi-hole on level energies of odd nuclei, particularly on level energies of quasi-particles lying near the Fermi level, is very remarkable, it changes transition probabilities, multipole moments, spectroscopic factors, and so on by less than 10% from values obtained by the usual calculations.

### ACKNOWLEDGMENT

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### REFERENCES

- (1) O. Nathan and S. G. Nilsson, Alpha-, Beta-, and Gamma-Ray Spectroscopy, ed. K. Siegbahn, North-Holland Publishing Co., Amsterdam (1965). Related references may be found there.
- (2) S. Yoshida, *Nucl. Phys.*, **38**, 380 (1962).
- (3) L. S. Kisslinger and R. A. Sorensen, *Rev. Mod. Phys.*, **35**, 853 (1963).
- (4) In the RPA scheme, one may also add the following terms:

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j' m' | jm) \alpha_{jm}^+ \{ \phi_{j_1j_1'}(-)^{\lambda-\mu} Q_{\lambda-\mu} + \varphi_{j_1j_1'} Q_{\lambda\mu}^+ \}$$

and

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j' m' | jm) (-)^{j'-m'} \alpha_{j'-m'} \{ \phi_{j_1j_1'}(-)^{\lambda-\mu} Q_{\lambda-\mu} + \varphi_{j_1j_1'} Q_{\lambda\mu}^+ \}$$

$$L_{jj':j_1j_1'} \equiv -\chi_\lambda (2\lambda+1/2j+1)^{1/2} \langle j_1' \| q_\lambda \| j' \rangle \langle j \| q_\lambda \| j_1 \rangle v_{j_1'j'} u_{jj_1} W(jj_1j'j_1'; \lambda\lambda)$$

to the right hand side of Eqs. 2a and 2b, respectively. They are numerically smaller than the terms including  $K_{jj'}$  and  $M_{jj'}$ . Here,  $\chi_\lambda$  represents a strength of the two body interaction of  $2^\lambda$ -pole. Explicit expressions for  $\varphi_{jj'}$  and  $\phi_{jj'}$  can be seen in Ref. 2.

- (5) Here  $q_\lambda \equiv i^\lambda w_\lambda Y_\lambda$ ,  $w_\lambda \equiv [(m\omega_0/\hbar)^{1/2} r]^\lambda$   
 $u_{j,j} \equiv U_j V_j + V_{j'} U_{j'}$ ,  $v_{j,j} \equiv U_j U_j - V_{j'} V_{j'}$

where  $r$  is the radial co-ordinate of nucleon,  $m$  being its mass. Harmonic oscillator wave functions, with angular frequency  $\omega_0$ , are used for single-particle wave functions.  $U_j$  and  $V_j$  mean the probability of orbit  $j$  being empty and occupied, respectively.

- (6) It is indispensable to take accounts of several major shells for an explanation of the anomalous-coupling states on the basis of the BCS method and RPA. H. Ikegami and M. Sano, *Phys. Lett.*, **21**, 323 (1966).
- (7) Half the solutions of the equations of motion, whose energies tend to those of a quasihole with zero, one, and two phonons in the limit of  $S_2^{-1/2} = 0$  are unphysical and must be rejected as in the RPA for even-even nuclei.
- (8) The result shown in Fig. 2 is essentially almost the same with those presented in Ref. 6.
- (9) For examples, low-lying levels having the same spin and parity are:  
 The ground and 0.265 MeV states;  $3/2^-$  in  $\text{As}^{75}$ .  
 The 0.249 and 0.439 MeV states;  $5/2^-$  in  $\text{Sc}^{77}$ .  
 The 0.131 and 0.279 MeV states;  $5/2^+$  in  $\text{Br}^{77}$ .  
 The ground and 0.261 MeV states;  $3/2^-$  in  $\text{Br}^{79}$ , etc.